Multiple-Period Attribution: Residuals and Compounding

Our reviewer gave these authors "full marks for dealing with an issue that performance measurers and vendors often regard as proprietary information." In 1994, Denis S. Karnosky, Ph.D. and Brian D. Singer, CFA wrote "Global Asset Management and Performance Attribution"*, a monograph that offers a performance attribution system for isolating the effects of market allocation, currency management, and security selection on global portfolios. In response to public inquiries about this method, the authors composed the following article, which addresses several issues, and describes the implementation of the methodology over multiple periods.*

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The Karnosky-Singer monograph[1] introduces an analytical framework designed to overcome the deficiencies in single-country attribution models. Its usefulness in attributing a global portfolio manager's returns to market, security, and currency management has been proven through implementation and analysis of performance. Although the monograph's theory is soundly based, in practice it sometimes fails to fully attribute all of a portfolio's total added value to the decision-making areas. This failure to fully explain the sources of added value is not due to a flaw in the theory, but rather is a result of at least four interrelated factors: 1) cross-products, 2) the simplifying assumptions made when collecting performance data and definitions of decision-making areas, 3) intramonth changes in portfolio strategy, and 4) the impact of compounding the returns across time.

RESIDUAL EFFECTS

These factors in concert create a residual, which is the difference between the portfolio's actual added value and the sum of the added values from market, security and currency selection. Over time, hopefully the re-

sidual returns from intramonth strategy changes will add value, while all other sources of residuals wash out to leave a clear picture of how the portfolio has performed. Through many years of implementation, it has been found that linking monthly data in a specific manner produces analyses for longer periods with small residuals and allows for an accurate assessment of portfolio performance over any selected time period. This paper reviews a suggested method for linking single period analysis into a multiple-period analysis; this is non-trivial because the total added value compounds through time.

Equalities

Over any time period:
Total Added Value = Portfolio Return - Benchmark Return
And:
Total Added Value = Market Selection + Currency Selection + Security Selection + residual

CONSIDER THE SOURCE

It is helpful to consider the sources of single-period residuals within the framework before discussing multiple-period analysis. In practice, trade-offs must be made between the amount of data gathered to analyze portfolio total returns and the increased level of accuracy gained from performing this operation. Further, the framework has cross-products, or small slivers of added-value that fall between the explained returns. One example of such a cross-product is exchange rate gains, or losses, on the same period's local market gains, or losses.[2] (Note the market and currency cross-products are accounted for within the framework, and do not end up in the residual.)

DEFINE PERIODS

Also, the ability to explain added values within single periods is dependent on the definition of a period. Each single period analysis will tend to become more accurate as the periods are defined as shorter lengths of time (such as weekly versus monthly periods). This property can be observed within the market selection added value term by studying the single period equation.

MARKET SELECTION

MARKET SELECTION

$$= \begin{Bmatrix} \text{Active} \\ \text{Market} \\ \text{Weight} \end{Bmatrix} - \begin{Bmatrix} \text{Passive} \\ \text{Market} \\ \text{Weight} \end{Bmatrix} \times \begin{Bmatrix} \text{Passive Market} \\ \text{Return} \\ \text{Premium} \end{Bmatrix} - \begin{Bmatrix} \text{Index Market} \\ \text{Return} \\ \text{Premium} \end{Bmatrix}$$

$$= \sum_i \left\{ \left[w_i - \overline{w}_i \right] \times \left[\left(\overline{r}_i - \overline{c}_i \right) - \mathrm{RP} \right] \right\}$$

where,

$\underline{w_i}$ = weight of country i assets

$\underline{r_i}$ = return from the assets of country i, in local-currency terms

$\underline{c_i}$ = return from country i Eurodeposits

RP = aggregate passive benchmark local-currency return premium

* Note: Plain lower-case letters indicate portfolio weights and returns; letters with a bar over them indicate passive benchmark weights and returns.

Static

Applying static, beginning of the month weights for both the portfolio and benchmark generally works well for computing added values. The effectiveness depends on the relative weights during the month not deviating significantly from the beginning of the month snapshot, as the active weights of a portfolio depend directly on the portfolio manager's decisions. For example, within a month a manager can decide to make a strategic shift in market weights by reducing holdings in one market and increasing them in another. Using beginning of the month weights, the changes from the shift would not be reflected in the attribution's weights until the start of the following month. So the portfolio's market selection added value would be calculated based on the original strategy, meaning any gains or losses from the strategy change would end up in the residual until the start of the next month.

More frequent snapshots of the portfolio would correct for this problem by reducing the length of time until changes in the active weights enter into the attribution calculations (attribution weights would accurately model the actual weights as they change with time). Yet, the marginal improvement in accuracy that would come from using less dated weights (especially on a regular basis) would involve a very large increase in the data maintained, technological resources required, and time spent analyzing the attribution. Keeping monthly periods, while being somewhat flexible in selecting the date for each period's active weights, is more easily implemented, if a slightly less rigorous solution. For example, if the above described shift in assets occurred on the third day of a given month, it would probably be more accurate to use active weights from the fourth day, rather than the beginning of the month.

Compounding

When moving from single period returns analyses to analyses over more than a single period, it is necessary to consider the impact of compounding. Portfolios and their benchmarks compound independently through time; so added values, or differences in returns, are also approximately compounded through time. Compounding within attributions is necessary for simple U.S. balanced portfolios as well as global portfolios with hedging. Table A (*see page 25*) is an example of a multiple-period attribution analysis for a three nation global portfolio.

Each market added value area can be viewed as a collection of weighted risk premiums. So, by adding or subtracting specific weighted risk premium returns it is possible to construct any of the market decision making areas, as described by the following equations:

Market selection:

$$\sum_i \left\{ \left[w_i - \overline{w}_i \right] \times \left[\left(\overline{r}_i - \overline{c}_i \right) - \mathrm{RP} \right] \right\} = \sum_i \left\{ w_i \times \left[\overline{r}_i - \overline{c}_i \right] - \overline{w}_i \times \left[\overline{r}_i - \overline{c}_i \right] - w_i \times \left[\mathrm{RP} \right] + \overline{w}_i \times \left[\mathrm{RP} \right] \right\}$$

Security selection:

$$\sum_i \left\{ \overline{w}_i \times \left[\left(r_i - c_i \right) - \left(\overline{r}_i - \overline{c}_i \right) \right] \right\} = \sum_i \left\{ \overline{w}_i \times \left[r_i - c_i \right] - \overline{w}_i \times \left[\overline{r}_i - \overline{c}_i \right] \right\}$$

Market cross-product:

$$\sum_i \left\{ \left[w_i - \overline{w}_i \right] \times \left[\left(r_i - c_i \right) - \left(\overline{r}_i - \overline{c}_i \right) \right] \right\} = \sum_i \left\{ w_i \times \left[r_i - c_i \right] - \overline{w}_i \times \left[r_i - c_i \right] - w_i \times \left[\overline{r}_i - \overline{c}_i \right] + \overline{w}_i \times \left[\overline{r}_i - \overline{c}_i \right] \right\}$$

The first step toward a multiple-period attribution of market added-values is calculating the weighted risk premium returns for each market in every period. Then, the weighted risk premium returns represented in Table B (*see page 26*) are compounded over the periods within the multiple-period attribution.

Next, the resulting compounded risk premiums are added or subtracted as necessary to form the multiple period market explained added values as described in Table C (*see page 26*).

The procedure to calculate multiple-period added values for currency selection, hedge selection and currency cross-product is completely analogous to their market counterparts.[3] Finally, it is helpful to restate the equality that gives residuals. The total added value over any selected period equals the difference in compounded returns between a portfolio and its benchmark. This equality determines the multiple-period residual once the total multiple-period explained added value has been calculated.

CONCLUSION

The Karnosky-Singer attribution framework is structured to provide information at all levels of decision making. Linking the single-period analysis into a multiple-period attribution study is a powerful tool for

TABLE A
EXAMPLE OF A MULTIPLE-PERIOD ATTRIBUTION

PASSIVE BENCHMARK		Passive market weight ($\overline{w}_i$)	Passive market return ($\overline{r}_i$)	Passive hedge weight ($\overline{h}_i$)	Eurodeposit return ($\overline{c}_i$)	Exchange rate return ($\overline{e}_i$)	Index average risk premium (RP) $= \sum_i \overline{w}_i \times (\overline{r}_i - \overline{c}_i)$
Period 1	U.S.	40.00%	9.25%	N/A	5.25%	N/A	2.95%
	Germany	30.00%	7.00%	N/A	5.50%	-8.00%	2.95%
	Japan	30.00%	8.50%	N/A	5.50%	-13.00%	2.95%
Period 2	U.S.	40.00%	8.50%	N/A	4.75%	N/A	0.60%
	Germany	30.00%	6.00%	N/A	4.00%	5.00%	0.60%
	Japan	30.00%	-2.00%	N/A	3.00%	17.00%	0.60%
Period 3	U.S.	40.00%	-5.25%	N/A	4.75%	N/A	-3.85%
	Germany	30.00%	3.75%	N/A	4.25%	6.00%	-3.85%
	Japan	30.00%	4.50%	N/A	3.50%	2.00%	-3.85%

ACTIVE PORTFOLIO		Active market weight (w_i)	Active market return (r_i)	Active hedge weight (h_i)	Eurodeposit return (c_i)	Exchange rate return (e_i)
Period 1	U.S.	36.25%	10.00%	15.00%	5.25%	N/A
	Germany	38.50%	6.00%	0.00%	5.50%	-8.00%
	Japan	25.25%	8.75%	-15.00%	5.50%	-13.00%
Period 2	U.S.	32.00%	7.50%	34.00%	4.75%	N/A
	Germany	34.25%	6.75%	-6.00%	4.00%	5.00%
	Japan	33.75%	-4.00%	-28.00%	3.00%	17.00%
Period 3	U.S.	29.75%	-1.75%	14.00%	4.75%	N/A
	Germany	42.00%	5.25%	0.00%	4.25%	6.00%
	Japan	28.25%	4.00%	-14.00%	3.50%	2.00%

Table B
Weighed Risk Premiums

		$w_i \times [\bar{r}_i - \bar{c}_i]$	$\bar{w}_i \times [\bar{r}_i - \bar{c}_i]$	$w_i \times [\text{RP}]$	$\bar{w}_i \times [\text{RP}]$	$\bar{w}_i \times [r_i - c_i]$	$w_i \times [r_i - c_i]$
U.S.	Period 1	1.45%	1.60%	1.07%	1.18%	1.90%	1.72%
	Period 2	1.20%	1.50%	0.19%	0.24%	1.10%	0.88%
	Period 3	-2.98%	-4.00%	-1.15%	-1.54%	-2.60%	-1.93%
	Compounded Periods 1-3	-0.39%	-1.00%	0.10%	-0.14%	0.34%	0.63%
Germany	Period 1	0.58%	0.45%	1.14%	0.89%	0.15%	0.19%
	Period 2	0.69%	0.60%	0.21%	0.18%	0.83%	0.94%
	Period 3	-0.21%	-0.15%	-1.62%	-1.16%	0.30%	0.42%
	Compounded Periods 1-3	1.05%	0.90%	-0.30%	-0.10%	1.28%	1.56%
Japan	Period 1	0.76%	0.90%	0.74%	0.89%	0.98%	0.82%
	Period 2	-1.69%	-1.50%	0.20%	0.18%	-2.10%	-2.36%
	Period 3	0.28%	0.30%	-1.09%	-1.16%	0.15%	0.14%
	Compounded Periods 1-3	-0.66%	-0.32%	-0.15%	-0.10%	-1.00%	-1.42%

investment management by providing a means for critical review of the decision process over long periods of time. As the issues involved with multiple-period attribution analysis are not particular to any one framework, the explanation is drawn broadly. Still, the explanation lays out in detail an approach for handling the complexities of multiple-period attribution analysis for the benefit of analysts seeking to implement the Karnosky-Singer framework.

Table C
Multiple-Period Added Values

	Market selection	Security selection	Market cross-product	Total market added value
U.S	0.37%	1.34%	-0.32%	1.39%
Germany	0.35%	0.38%	0.13%	0.85%
Japan	-0.30%	-0.68%	-0.08%	-1.06%
Total	0.42%	1.04%	-0.27%	1.19%

Note: Due to rounding sums and totals may deviate from expected amounts.

ENDNOTES

[1] This monograph develops an analytical framework for evaluating global asset markets and uses that framework to construct a performance attribution system that isolates the effects of market allocation, currency management, and deep theoretical issues of asset pricing or optimal investment strategies but, rather, on the issue of developing useful measures of the market and currency components of global investment issues.

This work reflects the ongoing efforts with Brinson Partners to address practical issues in the management of glo-

bal portfolios. The analytical framework and the performance attribution system are integral parts of our investment process, and we believe that open discussion of these tools will enhance general understanding of global investment issues. The analysis provides a consistent framework for all who are involved in the evaluation of investment opportunities, Performance, and risks.

This monograph reflects the discussions and research of many Brinson Partners investment managers and analysts, to all of whom we owe a great debt. The presentation benefited particularly from the thoughts and arguments of Gary Brinson, Richard Carr, Khaled Salama, Raymond Chan and Norman Cumming. Ray Chan was also indispensable in solving the large number of technical issues involved in the performance attribution program. Robert Clarke was instrumental in the early development of the attribution program. We also thank the Research Foundation of the Institute of Chartered Financial Analysts, and AIMR, for their support and encouragement in preparing this monograph.

[2] For example, a foreign asset with a 10% local market return and a 5% exchange rate return would have a total return (measured in base currency) of 15.5 percent.

TOTAL RETURN = (1+10%)x(1+5%)-1 = 15.5% = LOCAL RETURN + FX RETURN + CROSS-PRODUCT

The local return and exchange rate returns are 10% and 5% respectively. The cross-product is 0.5%, the exchange rate gain on the additional market exposure obtained through the 10% local return.

[3] The Eurodeposit returns in base currency ($[\bar{c}_i + \bar{e}_i]$, C and $[c_i + e_i]$) are weighted by the currency weights ($[w_i + h_i]$ and $[\bar{w}_i + \bar{h}_i]$), compounded and assembled into the decision-making areas.